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$C=KD$, where K is a positive fraction in its lowest terms or a positive integer, and D is a positive integer as above determined.

The second condition is not independent of the first, because, if K' is substituted for $2K+1$ and $2D'$ for D , in the above set of relations, then we have

$$\frac{B+C}{B-C} = \frac{D'K' + D + D'K' - D'}{D'K' + D' - D'K' + D'} = \frac{2D'K'}{2D'} = K',$$

which is the second condition. Hence the set of values found above includes all the possible relations which make A , B , and C positive integers. The values of A and B may be interchanged, and A , B , and C may each be multiplied by a common factor without changing the value of the original ratio.

Also solved by A. H. Holmes.

184. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Mich.

Prove that $\frac{\pi}{12} = \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{8} + \tan^{-1} \frac{1}{30} + \tan^{-1} \frac{1}{112} + \dots$, where 2, 8, 30, 112..., is a recurring series with the recursion formula $u_n = 4u_{n-1} - u_{n-2}$.

Solution by the PROPOSER.

If we reduce $\sqrt{3}$ to a continued fraction, we get for the convergents,

$$\frac{1}{1}, \frac{2}{1}, \frac{5}{3}, \frac{7}{4}, \frac{19}{11}, \frac{26}{15}, \frac{71}{41}, \frac{97}{56}, \frac{265}{153}, \frac{362}{209}, \dots$$

The alternate convergents,

$$\frac{1}{1}, \frac{5}{3}, \frac{19}{11}, \frac{71}{41}, \frac{265}{153}, \dots$$

are formed by taking the ratios of the corresponding terms of the two recurring series

$$\begin{array}{l} 1, 5, 19, 71, 265, \dots \\ 1, 3, 11, 41, 153, \dots \end{array}$$

both having the same scale of relation $u_n = 4u_{n-1} - u_{n-2}$.

We find by the usual methods that the n th terms of the two series are

$$\frac{\alpha^n - \beta^n}{\alpha - \beta} + \frac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} = u_n + u_{n-1}, \text{ and } \frac{\alpha^n - \beta^n}{\alpha - \beta} - \frac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} = u_n - u_{n-1},$$

where α and β are the roots of the equation $x^2 - 4x + 1 = 0$.

Taking the differences of the \tan^{-1} function of the reciprocals of the convergents, we have

$$\tan^{-1} \frac{1}{1} - \tan^{-1} \frac{3}{5} = \tan^{-1} \frac{1}{4}$$

$$\tan^{-1} \frac{3}{5} - \tan^{-1} \frac{11}{19} = \tan^{-1} \frac{1}{64}$$

$$\tan^{-1} \frac{11}{19} - \tan^{-1} \frac{41}{71} = \tan^{-1} \frac{1}{900}$$

$$\tan^{-1} \frac{41}{71} - \tan^{-1} \frac{153}{265} = \tan^{-1} \frac{1}{12544}$$

$$\dots \dots \dots$$

$$\tan^{-1} \frac{u_n - u_{n-1}}{u_n + u_{n-1}} - \tan^{-1} \frac{u_{n+1} - u_n}{u_{n+1} + u_n} = \tan^{-1} \frac{1}{4u_n^2}$$

$$\dots \dots \dots$$

Adding, we have

$$\tan^{-1} \frac{1}{1} - \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12} = \tan^{-1} \frac{1}{2^2} + \tan^{-1} \frac{1}{8^2} + \tan^{-1} \frac{1}{30^2} + \dots$$

$$+ \tan^{-1} \frac{1}{(2u_n)^2} + \dots$$



PROBLEMS FOR SOLUTION.

ALGEBRA.

360. Proposed by CHARLES C. GROVE, Columbia University, New York.

A bridge club of 28 members has 27 meetings. There are 7 tables with 4 members at each table. Can the players be so arranged that at the end of the season (27 meetings) each member will have played *with* every other member one game and *against* every other member two games, one game meaning one meeting; and how?

361. Proposed by C. E. GITHENS, Ph. D., Wheeling, W. Va.

Find three integral values for $[-10 + 9\sqrt{-3}]^{1/3} + [-10 - 9\sqrt{-3}]^{1/3}$. A solution not involving a cubic is desired.